are charged there is an ambiguity in the method of introducing a cutoff. What is significant, however, is that the magnetic moment's coefficient depends upon $(M_\nu/m)^2$. With the present upper limit on the muon neutrino's mass the magnetic moment is only on the order of 10^{-12} Bohr magnetons. Unfortunately, the present experimental upper limit on Γ_{ν_μ} is about 10^{-8} Bohr magnetons.[3]

IV. HIGHER ORDER EFFECTS

The electromagnetic and weak interactions of vector bosons are unrenormalizable. Lee and Yang, and Lee[14] have developed a method of handling the leading divergences in the perturbation expansion of vector boson electrodynamics. The general effect of the technique is to replace the term $\ln(\Lambda/m)$ with a term $-\frac{1}{2}\ln\alpha$.[15]

The leading divergences encountered in a certain class of expressions involving semiweak vector boson interactions have been summed by Feinberg and Pais.[16] The general effect of this summation procedure is to redefine the coupling constant g and to replace m^2 with $3/4m^2$. Applying the above considerations to the computation of the leptonic magnetic moments we see that $\Delta\Gamma_\mu$ may be increased to about 10^{-8} muon magnetons, and that the upper limit on $\Delta\Gamma_{\nu_\mu}$ might be raised to about 10^{-11} Bohr magnetons.

V. CONCLUSIONS

The above results suggest that the effects of weak interactions upon the magnetic moments of the leptons would be difficult if not impossible to observe. In the case of the muon (and the electron) the strong interactions are of more significance than the weak interactions in altering the magnetic moments. For the electronic neutrino the small mass value permits only a negligible magnetic moment.

If the muon's neutrino is shown to have a relatively large mass, and the sensitivity of neutrino scattering experiments is increased significantly, the observation of magnetic moments induced by weak interactions might be possible. Regardless of the neutrino's mass, scattering experiments involving large momentum-transfers will be dominated by the form factor calculated by Bernstein and Lee.

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[14] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962); and T. D. Lee, Phys. Rev. **128**, 899 (1962).
[15] See Ref. 13.
[16] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963); Y. Pwu and T. T. Wu, Phys. Rev. **133**, B778 (1964); see also the remarks at the end of the paper by Bernstein and Lee.

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Kaon Decays and Pion Statistics*

H. C. VON BAEYER
Vanderbilt University, Nashville, Tennessee
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Selection rules for nonleptonic decays of kaons are given under different assumptions about the statistics of pions. It is found that the observed decay modes are compatible only with the assumption that pions are bosons.

SEVERAL authors[1–3] have recently emphasized that the statistics of elementary particles should be determined directly without appeal to the spin-statistics theorem. This is particularly important since the indistinguishability of identical particles does not by itself imply that particles can obey only Bose-Einstein or Fermi-Dirac statistics.[4] The possiblity that particles obey generalized statistics exists *a priori*. The purpose of this note is to present arguments that pions obey Bose-Einstein statistics.

Our arguments are based on selection rules for the nonleptonic decays of kaons. We make explicit use of the following assumptions:

I. Conservation of angular momentum.

II. *CP* conservation. This conservation law is the most probable mechanism inhibiting the two pion decay of K_2^0.[5]

III. The kaon spin is zero. Evidence for this assignment independent of the statistics of pions includes the

* Supported in part by the National Science Foundation.
[1] O. W. Greenberg and A. Messiah (private communication to W. G. Holladay).
[2] D. B. Lichtenberg, in *Proceedings of Athens Conference on Resonant Particles*, edited by B. A. Munir and L. J. Gallaher (Ohio University Press, Athens, Ohio, 1963), p. 152.
[3] R. Gatto, Phys. Letters **5**, 56 (1963).
[4] A. Galindo, A. Morales, and R. Nuñez-Lagos, J. Math. Phys. **3**, 324 (1962).
[5] R. G. Sachs, Ann. Phys. (N. Y.) **22**, 239 (1963).

TABLE I. Selection rules and allowed values of T for $K \to 2\pi$. Values of T forbidden by the $|\Delta T| = \frac{1}{2}$ rule are enclosed in square brackets.

Decay	Bose statistics	Generalized statistics	Fermi statistics
$\theta^+ \to \pi^+\pi^0$	[2]	1, [2]	1
$K_1^0 \to \pi^+\pi^-$	0, [2]	0, [2]	CP
$\to \pi^0\pi^0$	0, [2]	0, [2]	CP
$K_2^0 \to \pi^+\pi^-$	CP	1	1
$\to \pi^0\pi^0$	CP	CP	CP

uniform population of the Dalitz plot in τ decay, Adair analysis of the decay $K_1^0 \to \pi^+ + \pi^-$ from $\pi^- + p \to \Lambda^0 + K^0$,[6] and the complete polarization of the muon in $K \to \mu + \nu$.[7]

IV. The parity of a two-pion state is $P = (-1)^L$, where L is the relative orbital angular momentum. The charge symmetry of pion-nucleon interactions indicates that the intrinsic parities of charged pions are equal.

V. Different assumptions about the statistics of pions furnish addition relations among the quantum numbers of a two-pion system. The isotopic spin T is related to L by $(-1)^{L+T} = \pm 1$, where the upper and lower signs refer to Bose-Einstein and Fermi-Dirac statistics, respectively. For a neutral system this relation can be written $C = \pm(-1)^L$. If pions obey any type of more general statistics, including parastatistics, then no further restrictions on C or T hold for a two-pion system.[1]

TABLE II. Selection rules and lowest values of (L,l) for $K \to 3\pi$.

Decay	Bose statistics	Generalized statistics	Fermi statistics
$\tau^+ \to \pi^+\pi^+\pi^-$	(0,0)	(0,0)	(1,1)
$\to \pi^0\pi^0\pi^+$	(0,0)	(0,0)	(1,1)
$K_1^0 \to \pi^0\pi^0\pi^0$	CP	CP	CP
$\to \pi^+\pi^-\pi^0$	(1,1)	(0,0)	(0,0)
$K_2^0 \to \pi^0\pi^0\pi^0$	(0,0)	(0,0)	(1,1)
$\to \pi^+\pi^-\pi^0$	(0,0)	(0,0)	(1,1)

Selection rules based on these assumptions are listed in Tables I and II. The notation CP indicates that the decay is forbidden by the CP selection rule. In Table I allowed values of T are listed while in Table II the lowest values of (L,l) are entered for allowed decays, where L is the relative orbital angular momentum of the first two pions and l is the angular momentum of the third.

The possibility of Fermi statistics for pions can be eliminated in two ways. Holladay and Thomas[8] have shown that the assignment $L = l = 1$ in the decay $\tau^+ \to \pi^+\pi^+\pi^-$ is incompatible with experiment. From Table II it is clear that this rules out Fermi statistics. From Table I it is furthermore seen that the decay $K^0 \to \pi^0\pi^0$ would not occur under the assumption of Fermi statistics.

There remains the possibility that pions obey generalized statistics. Greenberg and Messiah[1] have pointed out that the $|\Delta T| = \frac{1}{2}$ rule would not inhibit the decay $\theta^+ \to \pi^+\pi^0$ under this assumption. However, since this rule is not unanimously supported by experiment [9]they do not consider their argument very convincing.

Table I furnishes a stronger argument. The decay $K_2^0 \to \pi^+\pi^-$ is forbidden by CP only if pions are bosons. Hence the existence of a long-lived K^0 particle is proof that pions do not obey generalized statistics.[10]

A more difficult test is suggested by Table II. The decay $K_1^0 \to \pi^+\pi^-\pi^0$ is severely inhibited by the angular momentum barrier only if pions are bosons.[11] Observation of the branching ratio $K_1^0 \to 3\pi / K_1^0 \to 2\pi$ should thus furnish a further test of the statistics of pions.

We wish to thank Professor Holladay with whom we had many valuable discussions.

[6] R. K. Adair and E. C. Fowler, *Strange Particles* (Interscience Publishers, Inc., New York, 1963), p. 23.

[7] C. A. Coombes, B. Cork, W. Galbraith, G. Lambertson, and W. Wenzel, Phys. Rev. **108**, 1348 (1957).

[8] W. G. Holladay and B. S. Thomas, Phys. Rev. **110**, 981 (1958).

[9] G. Alexander, S. P. Almeida and F. S. Crawford, Phys. Rev. Letters **9**, 69 (1962).

[10] Proferror D. B. Lichtenberg has pointed out in a private communication that this argument can be made without assumption IV above. If pions obey generalized statistics there exist two S waves in the $\pi^+\pi^-$ system with CP quantum numbers $+1$ and -1, respectively. In that case both $K_1^0 \to 2\pi$ and $K_2^0 \to 2\pi$ are allowed and our conclusion follows.

[11] A. Pais and S. B. Treiman, Phys. Rev. **106**, 1106 (1957).